MATH 147 QUIZ 4 SOLUTIONS

1. In the homework of Wednesday, September 18, you were asked to find the absolute maximum and absolute minimum of $f(x,y,z) = x^2 + xz - y^2 + 2z^2 + xy + 5x$ on the solid block whose coordinates satisfy $-5 \leq x \leq 0;\ 0 \leq y \leq 3;\ 0 \leq z \leq 2$. Find the absolute maximum and minimum values for $f(x,y,z)$ just on top of the box. (7 points)

   As we are just looking at the top of the box, this amounts to checking the boundary for absolute extrema. We set $z = 2$, and we now have

   $$f(x,y,2) = x^2 + 2x - y^2 + 8 + xy + 5x = x^2 + 7x + xy - y^2 + 8.$$

   First, we check this square for critical points along the surface. We find

   $$f_x = 2x + y + 7; \quad f_y = x - 2y.$$

   Setting these equal to zero gives us a critical point at $(-14/5, -7/5)$. However, y cannot be negative, so we have no critical points on our surface to check.

   Now we check the boundary of this square itself. First, set $x = 0$. Then we have $f(0,y,2) = -y^2 + 8$. The derivative of this is $-2y$, which has a critical point at $y = 0$. This is on the corner, so we check it later. For $x = -5$, we have $f(-5,y,2) = -y^2 - 5y - 2$. The derivative is $-2y - 5$, which has a critical point at $y = -5/2$. We note that this is outside the domain of y, so it is not a critical point on the top of the box. Set $y = 0$ to get $f(x,0,2) = x^2 + 7x + 8$, and note the derivative is $2x + 7$, which has a critical point at $x = -7/2$. This is in the domain, and so we observe that $f(-7/2,0,2) = -17/4$. For the last side, set $y = 3$ and we get the function $f(x,3,2) = x^2 + 10x - 1$, with derivative $2x + 10$ giving critical point at $x = -5$. This is on the corner and we check it later.

   Lastly, check the corners. $f(-5,0,2) = -2$; $f(0,0,2) = 8$; $f(-5,3,2) = -26$; $f(0,3,2) = -1$. Among all of the values we have collected, we note that -26 is the absolute minimum, and 8 is the absolute maximum.

2. Given $f(x,y,z)$, with $x = x(u,v),\ y = y(u,v),\ z = z(u,v),$ write a formula for $\frac{\partial f}{\partial v}$. (3 points)

   Using the chain rule, we have

   $$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial v}.$$